Radiation Spectroscopy of Gamma-Rays

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My reference is the book <u>Radiation detection</u> and <u>measurement, 4th ed</u> by Glenn F. Knoll. And in this arctile I will follow the structure of Chapter 10 in the book and discuss the critical mechanisms of gamma-ray spectroscopy with scintillators.

1. Introduction

X-ray or gamma-ray photons are uncharged and therefore primarily invisible to the detector. The key of detecting them is to first convert incident photons into fast electrons and then collect these electrons.

Since the stopping power of gases is low, e.g. 1 MeV electron in STP gases can penetrate for several meters and most gamma-ray incuded pulses from a gas-filled counter actually arise from the solid counter wall and have lost varialbe energy in the wall, gas-filled detectors are not suitable for gamma-ray spectroscopy.

The thallium-activated sodium iodine scintillation detector arose in the 1950s have proven the effectiveness of scintillation detectors in gamma-ray spectroscopy.

2. Review on Gamma-Ray Interactions

In Chapter 2 we have studied the three interaction mechnisms of significance in gamma-ray spectroscopy. The dominant parameters in these three interactions are $h\nu$, the photon' s energy and *Z*, the atomic number of the absorbing material.



Figure 1: Relative importance of three main gamma-ray interactions

2.1. Photoelectric Absorption

Photoelectric absorption is an interaction that absorbs the photon and emits a photoelectron of the energy:

$$E_{e^-} = h\nu - E_b \tag{1}$$

Where E_b is the binding energy and is in the form of a characteristic X-ray or Auger electron¹. Most possibly, photoelectron emerges from the K shell, whose typcial binding energy ranges from few keV (₁₁Na, 1keV) to hundred keV(₈₈Ra, 100keV).

2.2. Compton Sacttering

The result of Compton scattering is a scattered photon and a recoil electron.

$$\begin{split} h\nu' &= \frac{h\nu}{(1+(h\nu)/m_0c^2)(1-\cos\theta)}\\ E_{e^-} &= h\nu-h\nu' \end{split} \tag{2}$$

The angular distribution of scattered gamma rays is predicted by the Klein-Nishina formula and is ploted polarly in Figure 2.

$$\begin{aligned} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= Zr_0^2 \bigg(\frac{1}{1 + \alpha(1 - \cos\theta)} \bigg)^2 \bigg(\frac{1 + \cos^2\theta}{2} \bigg) \\ & \left(1 + \frac{\alpha^2(1 - \cos\theta^2)}{(1 + \cos^2\theta)(1 + \alpha(1 - \cos\theta))} \right) \end{aligned}$$

$$\alpha = h\nu/m_0c^2, r_0 = \text{classical electron radius}$$
 (3)

¹Auger electrons have extremly short range because of their low energy



Compton edge Klein-Nishina formula Figure 2: Compton sacttering

And generally the shape of the electron energy distribution has a so-called Compton edge.

2.3. Pair Production

The process occurs in the intense electric field near the protons in the nuclei of the absorbing material and corresponds to the creation of an electron-positron pair at the point of complete disappearance of the incident gamma-ray photon.

$$E_{e^-} + E_{e^+} = h\nu - 2m_0c^2 \tag{4}$$

Once the kinetic energy of the positron is lost, it will annihilate with or combine with a normal electron in the absorbing medium, which will make the spectroscopy analysis more complicated as we shall see later.

3. Predicted Response Function

We have discussed the three main mechanisms of gamma-ray interactions. Their simplest energy spectrum of their own is presented in Figure 3.



Figure 3: Three main mechanisms in gammaray interactions

3.1. Ideal Case of Intermediate Detectors

As for practical detectors, the actual response is much more complicated. All these interactions take place and the properties of the detector material and the geometry of the detector will affect the response function as well as the cirumstances.

Figure 4: Small detector VS extreme large detector

As stated in Figure 4, the spectrum depends on whether all primary and Secondary interactions happen within the active volume of the detector or not.

The real detectors are all of medium size and we can never achieve the ideal case of all interactions happening within the active volume since there are always interactions near the entrance surface.

Figure 5: Real detector

It's worth mentioning that there is "Multiple Compton events". The multiple Compton events arise from the escape of the final scattered photons, which can thus partially fill in the gap between the Compton edge and the photopeak.

3.2. Complications in the Real Response

There are several more complicated cases than those shown in Figure 5.

3.2.1. Secondary Electron Escape

Especially for high energy gamma-rays, the Secondary electrons may escape the active volume since their range is larger than the size of the detector. This effect will alter the shape of the Compton continuum and lower the photofraction due to envents loss from the photopeak.

3.2.2. Bremsstrahlung Escape

Radiative processes for charged particles due to coulomb interactions are Bremsstrahlung, where energy converts into electromagnetic radiation. The linear specific energy loss is:

$$-\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_r = \frac{NEZ(Z+1)e^4}{137m_0^2c^4} \bigg(4\ln\frac{2E}{m_0c^2} - \frac{4}{3}\bigg)~(5)$$

Again for high energy (few MeV and above) gamma-rays and high atomic number absorber case, the Bremsstrahlung loss is of importance.

3.2.3. Characteristic X-ray Escape

Normally characteristic X-ray emission is reabsorbed near where it is produced. But if the photoelectric absorption occurs near a surface of the detector, the X-ray may escape and so does its energy. This gives rise to a new peak below the full energy photopeak. Usually we can only distinguish the K-shell X-ray escape peak from the photopeak.

3.2.4. Secondary Radiations Created Near the Source

3.2.4.1. Annihilation Peak

If the gamma-ray source happens to emit positrons, a peak of 0.511 MeV will rise since the positron will annihilate when stopped in the surrounding covering. Sometimes if the detector can detect both annihilation photons simutaneously, a peak of 1.022 MeV will appear.

3.2.4.2. Bremsstrahlung

Most commonly available gamma-ray sources are β^- emitters at the same time. And Bremsstrahlung photons (low energy most possibly) will contribute to the spectrum due to absorption of β^- decays in the encapsulation. So use of low atomic number absorbers will help minimize the generationn of Bremsstrahlung.

3.2.5. Effects of Surrounding Materials

3.2.5.1. Backscatterd Gamma-Rays

As discussed before, the energy of backscattered photon is:

$$h\nu'|_{\theta=\pi} = \frac{h\nu}{1+2h\nu/m_0 c^2}$$
(6)

And in the limit of $h\nu \gg m_0 c^2$, the energy reduces to $m_0 c^2/2$. The backscatter peak always occurs at 0.255 MeV or less.

Figure 7: Backscatterd gamma-rays

3.2.5.2. Other Secondary Radiations

Other interaction of the primary gamma-rays in the surrounding materials, for example, characteristic X-rays, will reach the detector and produce noticeable peaks. One method to reduce the effect is to use graded shields consisting of high-Z bulk and low-Z inner layers. The inner layers will absorb the strong characteristic X-rays from the bulk and only emit weak X-rays on their own.

Another effect of high energy primary gammarays is the enhanced pair production process within high-Z surrounding materials.

3.2.6. Coincidence Methods in Gamma-Ray Spectrometers

To achieve ideal delta response function, some steps are taken at the price of added complexity. For the case of sodium iodide spectrometers, the most common methods involve the use of an annular detector surrounding the primary crystal for Compton suppression by anticoincidence.

And here is an example from germanium detectors in Figure 8, coincidence detection of

the escaping photons in a surrounding annular detector(BGO and NaI(Tl)²)

(a):unsupressed

(b): suppressed

BGO and NaI(Tl) scintillators combined Compton suppression system Figure 8: Anticoincidence Compton Suppression

4. Properties of Scintillation **Gamma-Ray Spectrometer**

4.1. Response Function

Sodium iodide (NaI) gained popularity due to the high atomic number (Z = 53) of its iodine component, which enhances photoelectric absorption, resulting in high intrinsic detection efficiency and a large photofraction. This combination has led to the success of NaI scintillation spectrometers. Despite newer scintillators like LaBr3(Ce) offering higher light yield and better energy resolution, NaI(Tl) remains widely used due to its balance of low cost, availability, and adequate performance. Detailed studies and extensive experimental data on NaI(Tl) have further solidified its reliability and predictability in gamma-ray spectroscopy. NaI(Tl) also shows significantly better energy resolution compared to materials like BGO (see to Figure 9).

Figure 9: Comparative pulse height spectra measured for BGO and NaI

Due to the challenges in experimentally measuring the gamma-ray response function across all energies, calculations using the Monte Carlo method (see Figure 10) are essential, as they effectively model complex interactions within detectors. Additionally, for organic scintillators, which have low photoelectric interaction probabilities, deconvolution techniques can enhance gamma-ray spectra analysis (see Figure 10). These methodologies underscore the critical role of advanced computational and analytical techniques in gamma-ray spectroscopy.

deconvolution methods line: Monte Carlo Figure 10: Anticoincidence Compton Suppression

4.2. Energy Resolution

The energy resolution *R* is defined as

$$R = \frac{\rm FWHM}{H_0}$$
 where $H_0 =$ mean pulse height (7)

As argued in Chapter 4, the finite energy resolution of any detector may contain contributions resulting from the effects of charge collection statistics, electronic noise, variations in the detector response over its active volume, and drifts in operating parameters over the course of the measurement.

²BGO has the strong advantage that its high density and atomic number allow a more compact configuration compared with a sodium iodide detector of the same detection efficiency.

Howerver, the charge collection statistics, i.e. photoelectron statistics is the dominant factor in the energy resolution of scintillation detectors.

Therefore

$$R = \frac{\rm FWHM}{H_0} = K \frac{\sqrt{E}}{E} \propto \frac{1}{\sqrt{E}} \eqno(8)$$

If we take logarithm of both sides, we derive

$$\ln R = \ln K - \frac{1}{2} \ln E \tag{9}$$

And a more adequate representation of measured data takes the form

$$R = \frac{\sqrt{\alpha + \beta E}}{E} \tag{10}$$

where α , β are constants particular to any specific scintillator-PMT combination.

Figure 11: Measured $\ln R$ vs $\ln E$ from a NaI(Tl) detector

There are still other factors that affect the energy resolution, which include:

- variations in light generation and measurement
- \cdot non proportionality of light yield
- · long term drift

4.3. Engergy Calibration

Perfect proportionality between light output and deposited energy in scintillators would yield a linear calibration of pulse height or centroid channel number for full-energy peaks vs gamma-ray energy. However, due to inherent nonproportionality in scintillator responses to fast electrons, calibrations typically exhibit some nonlinearity, resulting in curved peak positions specific to each detector.

But normally the assumption of linearity leads to negligible error.

representation of the possible origins of electrons and photons³ measured light output per unit deposited energy for Nal (Tl), normalized to unity at 88 keV

Figure 12: Complex energy origin and non linearity response

The measured relative light output from sodium iodide over the low energy range is presented in Figure 12. There is a dip near the K-shell absorption edge of iodine. The response of the scintillator actually depends on the energy that is deposited by secondary electrons produced by the incident photon, and a complex mix of photoelectrons and/or Auger electrons will result from various types of photon interactions.

4.4. Detection Efficiency

Common application of sodium iodide scintillators is to measure the absolute intensity, which requires a prior knowledge of the effeiency of radiation detector. And published data on the detection efficiency of NaI(Tl) detectors are undoubtedly abundant.

³following the photoelectric absorption of an incident X-ray or gamma ray with energy *E* that is above the K-shell binding energy of 33.17 keV

Figure 13: Absorption efficiency of NaI of different thicknesses

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Appendix Periodic Table

1	1 1 1.0080 H Hydrogen Nonmetal	2											13	Puk			17	18 2 4.00260 Helium Noble Gas
2	3 7.0 Li Lithium Alkali Metal	4 9.012183 Be Beryllium Alkaline Earth Me			Atomic N	Name	.7 35.4 Cl Chlorine	Symb	c Mass, u ool nical Group	Block			5 10.81 B Boron Metalloid	6 12.011 C Carbon Nonmetal	7 14.007 N Nitrogen Nonmetal	8 15.999 O Oxygen Nonmetal	9 18.9984 F Fluorine Halogen	10 20.180 Neon Noble Gas
З	11 22.989 Na Sodium Alkali Metal	12 24.305 Mg Magnesium Alkaline Earth Me	3	4	5	6	7	8	9	10	11	12	13 26.981 Al Aluminum Post-Transition M	14 28.085 Silicon Metalloid	15 30.973 P Phosphorus Nonmetal	16 32.07 Sulfur Nonmetal	17 35.45 Cl Chlorine Halogen	18 39.9 Ar Argon Noble Gas
4	19 39.0983 K Potassium Alkali Metal	20 40.08 Ca Calcium Alkaline Earth Me	21 44.95591 SC Scandium Transition Metal	22 47.867 Ti Titanium Transition Metal	23 50.9415 V Vanadium Transition Metal	24 51.996 Cr Chromium Transition Metal	25 54.93804 Mn Manganese Transition Metal	26 55.84 Fe Iron Transition Metal	27 58.93319 CO Cobalt Transition Metal	28 58.693 Ni Nickel Transition Metal	29 63.55 Cu Copper Transition Metal	30 65.4 Zn Zinc Transition Metal	31 69.723 Ga Gallium Post-Transition M	32 72.63 Ge Germanium Metalloid	33 74.92159 As Arsenic Metalloid	34 78.97 Se Selenium Nonmetal	35 79.90 Br Bromine Halogen	36 83.80 Kr Krypton Noble Gas
5	37 85.468 Rb Rubidium Alkali Metal	38 87.62 Sr Strontium Alkaline Earth Me	39 88.90584 Y Yttrium Transition Metal	40 91.22 Zr Zirconium Transition Metal	41 92.90637 Nb Niobium Transition Metal	42 95.95 Mo Molybdenum Transition Metal	43 96.90636 Tc Technetium Transition Metal	44 101.1 Ru Ruthenium Transition Metal	45 102.9055 Rh Rhodium Transition Metal	46 106.42 Pd Palladium Transition Metal	47 107.868 Ag Silver Transition Metal	48 112.41 Cd Cadmium Transition Metal	49 114.818 In Indium Post-Transition M	50 118.71 Sn Tin Post-Transition M	51 121.760 Sb Antimony Metalloid	52 127.6 Te Tellurium Metalloid	53 126.9045 Iodine Halogen	54 131.29 Xe Xenon Noble Gas
6	55 132.90 CS Cesium Alkali Metal	56 137.33 Ba Barium Alkaline Earth Me		72 178.49 Hf Hafnium Transition Metal	73 180.9479 Ta Tantalum Transition Metal	74 183.84 W Tungsten Transition Metal	75 186.207 Re Rhenium Transition Metal	76 190.2 OS Osmium Transition Metal	77 192.22 Ir Iridium Transition Metal	78 195.08 Pt Platinum Transition Metal	79 196.96 Au Gold Transition Metal	80 200.59 Hg Mercury Transition Metal	81 204.383 TI Thallium Post-Transition M	82 207 Pb Lead Post-Transition M	83 208.98 Bi Bismuth Post-Transition M	84 208.98 PO Polonium Metalloid	85 209.98 At Astatine Halogen	86 222.01 Rn Radon Noble Gas
7	87 223.01 Fr Francium Alkali Metal	88 226.02 Ra Radium Alkaline Earth Me		104 267.1 Rf Rutherfordium Transition Metal	105 268.1 Db Dubnium Transition Metal	106 269.1 Sg Seaborgium Transition Metal	107 270.1 Bh Bohrium Transition Metal	108 269.1 HS Hassium Transition Metal	109 277.1 Mt Meitnerium Transition Metal	110 282.1 DS Darmstadtium Transition Metal	111 282.1 Rg Roentgenium Transition Metal	112 286.1 Cn Copernicium Transition Metal	113 286.1 Nh Nihonium Post-Transition M	114 290.1 Fl Flerovium Post-Transition M	115 290.1 Mc Moscovium Post-Transition M	116 293.2 LV Livermorium Post-Transition M	117 294.2 TS Tennessine Halogen	118 295.2 Og Oganesson Noble Gas
				57 138.9055 La Lanthanum Lanthanide	58 140.116 Ce Cerium Lanthanide	59 140.90 Pr Praseodymium Lanthanide	60 144.24 Nd Neodymium Lanthanide	61 144.91 Pm Promethium Lanthanide	62 150.4 Samarium Lanthanide	63 151.964 Eu Europium Lanthanide	64 157.2 Gd Gadolinium Lanthanide	65 158.92 Tb Terbium Lanthanide	66 162.500 Dy Dysprosium Lanthanide	67 164.93 Ho Holmium Lanthanide	68 167.26 Er Erbium Lanthanide	69 168.93 Tm Thulium Lanthanide	70 173.05 Yb Ytterbium Lanthanide	71 174.9668 Lu Lutetium Lanthanide
				89 227.02 Actinium Actinide	90 232.038 Th Thorium Actinide	91 231.03 Pa Protactinium Actinide	92 238.0289 U Uranium Actinide	93 237.04 Np Neptunium Actinide	94 244.06 Pu Plutonium Actinide	95 243.06 Am Americium Actinide	96 247.07 Cm Curium Actinide	97 247.07 Bk Berkelium Actinide	98 251.07 Cf Californium Actinide	99 252.0830 ES Einsteinium Actinide	100 257.0 Fm Fermium Actinide	101 258.0 Md Mendelevium Actinide	102 259.1 No Nobelium Actinide	103 266.1 Lr Lawrencium Actinide

Figure 14: Periodic Table

Terminologies

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I refered to <u>電機工程學術名詞</u> for the translation of some terms.

ABBR.	Term	术语		
	attenuation	衰减		
	deposit	沉积		
STP	standard temperature and pressure	标准状况		
	recoil	反冲		
	photopeak	光电峰		
	photofraction	峰总比		
	encapsulation	封装		
	graded shield	梯度屏蔽		
	true coincidence	真符合		
	chance coincidence	偶然符合		
	sum peak	和峰		
BGO	bismuth germanate oxide	锗酸铋		
	scintillation efficiency	闪烁效率		

Table 1: Terminologies