Special Relativity

1. The Special Theory of Relativty

Def 1 **Minowski Metric**:

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \tag{1}$$

Def 2: For the scalar product of a 4-vector with itself $a^\mu a_\mu$

spacelike $a^\mu a_\mu > 0$

timelike $a^\mu a_\mu < 0$

lightlike $a^\mu a_\mu = 0$

Def 3 **Lorentz Transform Matrix**:

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & -\gamma\beta & & \\ -\gamma\beta & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \tag{2}$$

$$x_\mu = g_{\mu\nu} x^\nu \tag{3}$$

$$\overline{x^\mu} = \Lambda^\mu{}_\nu x^\nu \tag{4}$$

Def 4 **Proper Time**:

$$\mathrm{d}\tau = \sqrt{1 - \frac{u^2}{c^2}}\,\mathrm{d}t \tag{5}$$

Def 5 **Proper Velocity**:

$$\eta^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} \tag{6}$$

$$\eta^0 = c\frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{c}{\sqrt{1 - u^2/c^2}} \tag{7}$$

2. Relativistic Mechanics

Def 6 **Energy**:

$$E = p^0 c = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{8}$$

Def 7 **Momentum**:

$$\vec{p} = m\vec{\eta} = m(\eta^1, \eta^2, \eta^3) \tag{9}$$

Def 8 **Minowski Force**:

$$K^\mu = \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} \tag{10}$$

Thm 9 **Conservation of Energy and Momentum**:

$$p^\mu p_\mu = -m^2 c^2 \tag{11}$$

$$E^2 - p^2 c^2 = m^2 c^4 \tag{12}$$

Def 10 **Lorentz Invariant**: An Lorentz invariant is the same value in all inertial systems.

Proper time τ and γL are both Lorentz invariants.