Special Relativity

The Special Theory of Relativty

Definition 1 (**Minowski Metric**):

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Definition 2: For the scalar product of a 4-vector with itself $a^\mu a_\mu$

spacelike $a^\mu a_\mu > 0$
timelike $a^\mu a_\mu < 0$
lightlike $a^\mu a_\mu = 0$

Definition 3 (**Lorentz Transform Matrix**):

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & -\gamma\beta & & \\ -\gamma\beta & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$x_\mu = g_{\mu\nu} x^\nu$$

$$\overline{x^\mu} = \Lambda^\mu{}_\nu x^\nu$$

Definition 4 (**Proper Time**):

$$\mathrm{d}\tau = \sqrt{1 - \frac{u^2}{c^2}}\,\mathrm{d}t$$

Definition 5 (**Proper Velocity**):

$$\eta^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}$$

$$\eta^0 = c\frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{c}{\sqrt{1 - u^2/c^2}}$$

Relativistic Mechanics

Definition 6 (**Energy**):

$$E = p^0 c = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Definition 7 (**Momentum**):

$$\vec{p} = m\vec{\eta} = m(\eta^1, \eta^2, \eta^3)$$

Definition 8 (**Minowski Force**):

$$K^\mu = \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau}$$

Theorem 1 (**Conservation of Energy and Momentum**):

$$p^\mu p_\mu = -m^2 c^2$$

$$E^2 - p^2 c^2 = m^2 c^4$$

Definition 9 (**Lorentz Invariant**): An Lorentz invariant is the same value in all inertial systems.

Proper time τ and γL are both Lorentz invariants.